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Weak-Compatible Mappings in Intuitionistic Fuzzy 3- Metric Spaces

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Abstract—*In the present paper, our aim is to prove a common fixed point theorem for four mappings in intuitionistic fuzzy-3 metric space.*

1. INTRODUCTION

In 1965, the concept of fuzzy sets was introduced by Zadeh [16]. Many authors have introduced the concept of fuzzy metric space in different ways

([3], [4], [7], [9]). George and Veeramani [4] modified the concept of fuzzy metric space introduced by Kramosil and Michalek [9] and defined a Hausdorff topology on this fuzzy metric space. Jungck [8] gave the more generalized concept compatibility than commutativity and weak commutativity in metric space and proved common fixed point theorems. Singh and Chauhan [14] introduced the concept of compatibility in fuzzy metric space and proved some common fixed point theorems in fuzzy metric spaces in the sense of George and Veeramani [4]. Park [11] using the idea of intuitionistic fuzzy sets, defined the notion of intuitionistic fuzzy metric spaces with the help of continuous t-norm and continuous t-conorm as a generalization of fuzzy metric space due to George and Veeramani [4].Recently, Chauhan and Singh[1] proved a fixed point theorem in intuitionistic fuzzy-3 metric space. The purpose of this paper is to prove a fixed point theorem in intuitionistic fuzzy-3 metric space through weak compatibility.

2. PRELIMINARIES

Definition 1. A binary operation

$* : [0, 1]^4 \to [0, 1]$ is called a continuous t-norm if $([0, 1], *)$ is an Abelian topological monoid with the unit 1 such that $a_1 * b_1 * c_1 * d_1 \leq a_2 * b_2 * c_2 * d_2$ whenever $a_1 \leq a_2$, $b_1 \leq b_2$, $c_1 \leq c_2$ and

$d_1 \leq d_2$ for all $a_1, a_2, b_1, b_2, c_1, c_2$ and

$d_1, d_2 \in [0, 1]$.

Definition 2. A binary operation

$\lozenge : [0, 1]^4 \to [0, 1]$ is called a continuous t-norm if $([0, 1], \lozenge)$ is an abelian topological monoid with the unit 1 such that

$a_1 \lozenge b_1 \lozenge c_1 \lozenge d_1 \leq a_2 \lozenge b_2 \lozenge c_2 \lozenge d_2$ whenever $a_1 \leq a_2$, $b_1 \leq b_2$, $c_1 \leq c_2$ and

$d_1 \leq d_2$ for all $a_1, a_2, b_1, b_2, c_1, c_2$ and

$d_1, d_2 \in [0, 1]$.

Definition 3. A 5-tuple $(X, M, N, *, \lozenge)$ is called a intuitionistic fuzzy 3-metric space if X is an arbitrary (non-empty) set, $*$ is a continuous t-norm, $\lozenge$ a continuous t-conorm and M, N are intuitionistic fuzzy sets on $X^4 \times [0, \infty)$, satisfying the following conditions :for all x, y, z u, $v \in X$ and $t_1, t_2, t_3, t_4 > 0$

(FM'' 1)$M(x, y, z, u, t)+N(x, y, z, u, t) \leq 1$

(FM''–2)$M(x, y, z, u, 0) = 0$

(FM'' –3) $M(x, y, z, u, t) = 1$ for all $t>0$

when at least two of the three simplex

$\langle x, y, z, u \rangle$ degenerate

(FM''4)$M(x,y,z,u, t) = M(x,u,z,y,t)$

$=M(y, z, u, x, t)$

$=M(z, u, x, y, t) = \cdots$,

(FM''5)$M(x, y, z, u, t_1 + t_2 + t_3 + t_4)$

$\geq M(x, y, z, v, t_1) * M(x, y, v, u, t_2) *$

$M(x, v, z, u, t_3) * M(v, y, z, u, t_4)$,

(FM''6)$M(x, y, z, u, \cdot) : [0, \infty) \to [0, 1)$

is left continuous

(FM''–7) $\lim_{t\to\infty} M(x, y, z, u, t) = 1$ for

all x, y, z, $u \in X$ and $t > 0$,

(FM''–8) $N(x, y, z, u, 0) = 1$

(FM''–9)$N(x, y, z, u, t) = 0$ for all $t > 0$

only when the three simplex $\langle x,y,z,u \rangle$

degenerate

(FM''−10) N(x,y,z,u, t) = N(x, u, z, y, t) = N(y, z, u, x, t) = N(z, u, x, y, t) = ···,

(FM''−11)$N(x, y, z, u, t_1 + t_2 + t_3 + t_4)$

$\leq N(x, y, z, v, t_1) \lozenge N(x, y, v, u, t_2) \lozenge$

$N(x, v, z, u, t_3) \lozenge N(v, y, z, u, t_r)$,

(FM''−12)N(x, y, z, u, ·) : $[0, \infty) \to [0, 1]$ is right continuous,

(FM''−13) $\lim_{t\to\infty} N(x, y, z, u, t) = 1$ for all x, y, z, u∈X and t > 0.

Definition 4. Let (X, M, N, ∗, ◊) be an intuitionistic fuzzy 3-metric space.

(a) A sequence $\{x_n\}$ in X is said to be convergent to a point x∈X if

$\lim_{n\to\infty} M(x_n, x, a, b, t) = 1$,

and $\lim_{n\to\infty} n(x_n, x, a, b, t) = 0$ for all a, b ∈X with t > 0.

A sequence $\{x_n\}$ in X is called Cauchy sequence if $\lim_{n\to\infty} M(x_{n+m}, x_n, a, b, t) = 1$, $\lim_{n\to\infty} n(x_{n+m}, x_n, a, b, t) = 0$ for all a, b∈X,

t > 0.

(b) An intuitionistic fuzzy 3-metric space in which every Cauchy sequence is convergent is said to be complete.

Definition5. Two maps A and B from an intuitionistic fuzzy 3-metric space

(X, M, N, ∗, ◊) into itself are said to be compatible if

$\lim_{n\to\infty} M(ABx_n, BAx_n, a, b, t) = 1$,

$\lim_{n\to\infty} N(ABx_n, BAx_n, a, b, t) = 0$

for all a, b∈X and t > 0, whenever $\{x_n\}$ is a sequence in X such that

$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = x \in X$.

Lemma1. In a intuitionistic fuzzy 3- metric space (X, M, N, ∗, ◊) for all x, y, z∈X, M(x, y, ·), N(x, y, ·) are non-decreasing and non-increasing functions respectively.

Remark1. Since ∗, ◊ are continuous, it follows from (FM5, FM11) that the limit of a sequence in an intuitionistic fuzzy metric 3-space is unique.

Definition 6. Self- mappings A and B of an intuitionistic fuzzy metric space

(X,M, N, *, ◊) is said to be weakly compatible if ABx = BAx when Ax = Bx for some x ∈ X.

3. MAIN RESULT

Theorem: Let A, B, S and T be self-maps of a intuitionistic fuzzy 3-metric space (X, M, N, ∗, ◊) satisfying

$A(X) \subset T(X)$ and $B(X) \subset S(X)$ …(1) and

$M(Ax, By, a, b, t) \geq r(M(Sx, Ty, a, b, t))$,

$N(Ax, By, a, b, t) \leq r'(N(Sx, Ty, a, b, t))$ …(2)

For all x, y, a, b ∈X, where

r : [0, 1]→[0, 1] and r′ : [0, 1]→[0, 1] is a continuous function such that r(t) > t and r′(t) < t for each 0 < t < 1.

Suppose that one of T(X) and S(X) is a complete subspace of X and the pairs

(A, S) and (B, T) are weakly compatible. Then, A, B, S and T have a unique common fixed point in X.

Proof. Let x_0 be an arbitrary point in X by (1) we can define inductively a sequence $\{y_n\}$ in X such that

$y_{2n} = Ax_{2n} = Tx_{2n+1}$ and

$y_{2n+1} = Sx_{2n+2} = Bx_{2n+1}$

for n = 0, 1, 2,….

Using in (2) we have

$M(y_{2n},y_{2n+1}, a, b, t)$

$= r(M(Ax_{2n}, Bx_{2n+1}, a, b, t))$

$\geq r(M(Sx_{2n}, Tx_{2n+1}, a,b,t))$,

$= r(M(y_{2n-1}, y_{2n}, a, b, t))$

$> M(y_{2n-1}, y_{2n}, a, b, t))$,

$N(y_{2n},y_{2n+1}, a,b,t)=r'(N(Ax_{2n},Bx_{2n+1}, a,b, t)) \leq r'(N(Sx_{2n}, Tx_{2n+1}, a, b, t))$,

$= r'(N(y_{2n-1}, y_{2n}, a, b, t))$

$< N(y_{2n-1}, y_{2n}, a, b, t))$.

Similarly,

$M(y_{2n+1}, y_{2n+2}, a,b,t) > M(y_{2n}, y_{2n+1}, a, b, t))$,

$N(y_{2n+1}, y_{2n+2}, a,b,t) < N(y_{2n}, y_{2n+1}, a, b, t))$.

Then

$M(y_n, y_{n+1}, a, b, t) > M(y_{n-1}, y_n, a, b, t))$,

$N(y_n, y_{n+1}, a, b, t) < N(y_{n-1}, y_n, a, b, t))$

Hence the sequence $\{M(y_n, y_{n+1}, a, b, t)\}$ is an increasing sequence of positive real numbers in [0, 1] and tends to a limit $\ell \geq 1$ and $\{N(y_n, y_{n+1}, a, b, t)\}$ is an decreasing sequence of positive real numbers in [0, 1] and tends to a limit $\ell \leq 1$. If $\ell < 1$, then

$\lim_{n\to\infty} M(y_{n+1},y_n, a, b, t) = 1 > r(\ell) > 1$, $\lim_{n\to\infty} N(y_{n+1}, y_n, a, b, t) = 1 < r'(\ell) < 1$

which is a contradiction. So, $\ell = 1$ and

$\ell = 0$ respt.

Now, for any positive integer p

$M(y_n, y_{n+p}, a, b, t) \geq M(y_n, y_{n+1}, a, b, t/p) *$

$M(y_{n+1}, y_{n+2}, a, b, t/p) *\ldots*$

$M(y_{n+p-1}, y_{n+p}, a, b, t/p)$,

$N(y_n, y_{n+p}, a, b, t) \leq N(y_n, y_{n+1}, a, b, t/p) \Diamond$

$N(y_{n+1}, y_{n+2}, a, b, t/p) \Diamond\ldots\Diamond$

$N(y_{n+p-1}, y_{n+p}, a, b, t/p)$

Taking the limit as $n\to\infty$ we get,

$\lim_{n\to\infty} M(y_n, y_{n+p}, a,b,t) \geq 1 * 1 *\ldots* 1=1.$

$\lim_{n\to\infty} N(y_n, y_{n+p}, a,b,t) \leq 0 \Diamond 0 \Diamond\ldots\Diamond 0=0.$

So,

$\lim_{n\to\infty} M(y_n, y_{n+p}, a, b, t) = 1,$

$\lim_{n\to\infty} N(y_n, y_{n+p}, a, b, t) = 0.$

Therefore, $\{y_n\}$ is a Cauchy sequence in X. Then, the subsequence

$\{y_{2n}\} = \{Tx_{2n+1}\} \subset T(X)$ is a Cauchy sequence in T(X). Suppose that T(X) is complete. So $\{y_{2n}\}$ converges to a point

$z = Tv$ for some $v\in X$. Hence, the sequence $\{y_n\}$ converges also to z and the subsequences $\{Ax_{2n}\}$, $\{Bx_{2n+1}\}$, $\{Sx_{2n+2}\}$ and $\{Tx_{2n+1}\}$ converge to z.

If $z \neq Bv$, using (2) we get

$M(Ax_{2n}, Bv, a,b,t) \geq r(M(Sx_{2n},Tv, a, b, t)),$

$N(Ax_{2n}, Bv, a, b, t)\leq r'(N(Sx_{2n},Tv, a, b, t)).$

Letting $n\to\infty$ we obtain

$M(z, Bv, a, b, t) \geq r(M(z, z,a,b,t))=r(1) = 1,$

$N(z, Bv, a, b,t) \leq r'(N(z, z, a, b,t))=r(0)= 0.$

Therefore, $z = Bv = Tv$. Since

$B(X) \subset S(X)$, there exists $u\in X$ such that $Bv = Su = z$. If $z \neq Au$, using (2) we get

$M(Au, Bv, a, b, t) \geq r(M(Su, Tv, a, b, t)),$

$N(Au, Bv, a, b, t) \leq r'(N(Su, Tv, a, b, t)).$

Then,

$M(Au, z, a, b, t) \geq r(M(z, z, a, b, t)) = 1,$

$N(Au, z, a, b, t) \leq r'(N(z, z, a, b, t)) = 0.$

Therefore, $z = au = Su$. Since the pair

$\{A, S\}$ is compatible we have $SAu = ASu$, i.e., $Az = Sz$. If $z \neq Az$, using (2) we have

$M(Az,Bv,a,b,t)\geq r(M(Sz,Tv,a,b,t))= r(M(Az, z, a, b, t)) > M(Az, z, a, b, t),$

$N(Az,Bv,a,b,t)\leq r'(N(Sz,Tv,a,b,t))= r'(N(Az,z,a,b,t)) < N(Az, z, a,b,t)$

which is a contradiction. So, $z = Az = Sz$. If $z \neq Bz$, using (3.5) we get

$M(Az, Bz, a, b, t) \geq r(M(Sz, Tz, a, b, t)),$

$N(Az, Bz, a, b, t) \leq r'(N(Sz, Tz, a, b, t)).$

Then,

$M(z, Bz, a, b, t) = r(M(z, Bz, a, b, t))$

$> M(z, Bz, a, b, t),$

$N(z, Bz, a, b, t) = r'(N(z, Bz, a, b, t))$

$< N(z, Bz, a, b, t)$

Therefore, $z = au = Su$. Since the pair

$\{A, S\}$ is compatible we have $SAu = ASu$, i.e., $Az = Sz$. If $z \neq Az$, using (2) we have

$M(Az,Bv,a,b,t)\geq r(M(Sz,Tv, a, b, t))$

$= r(M(Sz, z, a,b,t)) > M(Az, z, a, b, t),$

$N(Az,Bv,a,b,t)\leq r'(N(Sz,Tv, a, b,t))$

$= r'(N(Az, z, a, b,t)) < N(Az, z, a, b,t)$

which is a contradiction. So, $z = Az = Sz$. If $z \neq Bz$, using (3.5) we get

$M(Az, Bz, a, b, t) \geq r(M(Sz, Tz, a, b, t)),$

$N(Az, Bz, a, b, t) \leq r'(N(Sz, Tz, a, b, t)).$

Then,

$M(z, Bz, a, b, t) = r(M(z, Bz, a, b, t))$

$> M(z, Bz, a, b, t);$

$N(z, Bz, a, b, t) = r'(N(z, Bz, a, b, t))$

$< N(z, Bz, a, b, t)$

which is a contradiction.

Hence, $z = Bz = Tz$. Therefore, z is a common fixed point of A, B, S and T. The uniqueness of z follows from (2).

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